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Vector bundles over projective spaces. The case $\mathbb{F}_1$

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Abstract. Over the field of one element, vector bundles over n-dimensional projective spaces are considered. It is shown that all line bundles are tensor powers of the Hopf bundle and all vector bundles are direct sums of line bundles. This is in complete analogy to the case of the projective line over an arbitrary classical field, but drastically simpler in comparison with projective spaces of higher dimensions.

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In recent years there has been quite a bit of activity concerning the field of one element, $\mathbb{F}_1$ (see e.g. [2], [6], [7] and [9]). By now, there are different versions in existence, so C. Soulé's paper [8] and a number of others [3–5,10], all of them interrelated with each other. We follow here the approach of Anton Deitmar in [4], which is closest to standard algebraic geometry.

The purpose of this paper is to study vector bundles over the projective line and, more generally, on n-dimensional projective space. As the classification of vector bundles on $\mathbb{P}^1$ is rather immediately related with the Euclidean algorithm, it was our hope that something combinatorially interesting might show up in this context, as the underlying group here is the symmetric group.

However it seems that the situation at least for this question is too simple. Over $\mathbb{P}^1$ our result is identical with Grothendiecks result that any vector bundle on $\mathbb{P}^1$ is a direct sum of line bundles. The latter are classified up to isomorphisms by their degree. The same holds true for vector bundles on $\mathbb{P}^n$ over $\mathbb{F}_1$, so here the situation is drastically simpler than in the classical case.

Formally, this project came up as the Diplom-thesis of the second author in (2008), written under guidance of the last and with crucial observations of the first author.

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1. The category of schemes on $\mathbb{F}_1$. We repeat shortly some of the concepts of Deitmar [4]. In analogy with the category of commutative rings, the category underlying the constructions here is the category of commutative monoids $(M, \cdot)$ with neutral element 1. The basic examples one should have in mind are the monoids $(\mathbb{N}, +)$ and $(\mathbb{Z}, +)$ where for our purposes $\mathbb{N}$ always contains 0 as the neutral element and where we write the composition on $M = \mathbb{N}$ resp. $M = \mathbb{Z}$ additively as usual. In connection with these one should think of the affine line $\mathbb{A}^1$ and its associated commutative ring $k[t]$ as the monoid $\{1, t, t^2, \ldots\} \subset k[t]$ as well as the multiplicative group $\mathbb{G}_m$ and its coordinate ring $k[t, t^{-1}]$ as the monoid of elements $\{1, t^{\pm 1}, t^{\pm 2}, \ldots\}$.

Definition 1.1. For a monoid $(M, \cdot)$ an ideal I is a subset $I \subsetneqq M$ satisfying $M \cdot I \subset I$. In particular, the empty set is an ideal in any monoid.

Definition 1.2. An ideal $p \subsetneqq M$ is prime if for $x, y \in M$ with $x \cdot y \in p$ we have $x \in p$ or $y \in p$.

The following is clear.

Lemma 1.3. *Let $I \subset M$ be an ideal. Then I is prime if and only if $(M \setminus I, \cdot)$ is a submonoid of $(M, \cdot)$.*

Definition 1.4. $\operatorname{Spec}(M) := \{p \subseteq M | p \text{ is prime}\}$ is the spectrum of the monoid $(M, \cdot)$.

Remark 1.5. For a monoid $(M, \cdot)$ the set $M^\times$ is the set of invertible elements in M. The set $(M \setminus M^\times)$ obviously is the unique maximal (prime) ideal M. In particular any monoid $(M, \cdot)$ in this sense corresponds even to a local ring.

Spec M will obtain a topology by taking as closed subsets the sets $V(I) := \{p \in \operatorname{Spec} M | p \supseteq I\}$ for any ideal or subset $I \subseteq M$. Special open subsets are the sets $D(f) := \{p \in \operatorname{Spec} M | f \notin p\}$ for an element $f \in M$. The open sets $D(f), f \in M$, form a basis of the topology.

Proposition 1.6. (i) *Let $(M, \cdot)$ be a monoid, $S \subseteq M$ a submonoid. Then there is a monoid $S^{-1}M$ and a homomorphism of monoids $\phi : M \to S^{-1}M$, unique up to unique isomorphism, satisfying the following property: $\phi(S)$ consists of invertible elements in $S^{-1}M$ and ϕ is universal with this property.*

(ii) $(S^{-1}M, \phi)$ *is unique up to unique isomorphism.*

Proof. Trivial. See also [4] □

Then, as explained in [4], there is a sheaf of monoids, $M^\sim$ on Spec M such that the stalk of $M^\sim$ at $p \in \operatorname{Spec} M$ is $M_p := S^{-1}M$ with $S = M \setminus p$. The pair $(\operatorname{Spec}(M), M^\sim)$ consisting of the topological space M and the sheaf of monoids $M^\sim$ is called an affine scheme (over $\mathbb{F}_1$).

For the convencience of the reader we recall the concept of a local morphism in this context. In general a homomorphism $\phi : (M, \cdot) \to (M', \cdot)$ is called local, iff $\phi^{-1}((M')^\times) = M^\times$ holds for the sets of units. Local morphisms between affine schemes (over $\mathbb{F}_1$) are morphisms between monoided spaces,

$$\tilde{\phi} : (M', \cdot)^\sim \to (M, \cdot)^\sim,$$

such that the occuring canonical homomorphisms $\tilde{\phi}_p : M_p \to M'_q$ with $\tilde{\phi}(q) = p, p \in \operatorname{Spec}(M), q \in \operatorname{Spec}(M')$, are local for all $q \in \operatorname{Spec}(M')$.

Proposition 1.7. *There is an antiequivalence of categories between the category of monoids with monoid homomorphisms and the category of affine schemes with local morphisms.*

Proof. See [4, Proposition 2.2]. □

Examples 1.8.
- $(M, \cdot) = (\mathbb{N}, +)$. Obviously, $\operatorname{Spec} \mathbb{N} = \{p, \eta\}$ where $p = \{1, 2, \ldots\}$ is the closed point and $\eta = \emptyset$ is the generic point. The stalks of $(\mathbb{N}, +)$ are $M_\eta = (\mathbb{Z}, +)$ and $M_p = M$. In particular, $(\mathbb{N}, +)$ is a local monoid, as are all monoids, with unique maximal ideal $p = M \setminus M^\times$.
- $(M, \cdot) = (\mathbb{N}^s, +)$. The prime ideals in $(\mathbb{N}^s, +)$ can be described as follows: Suppose $X \subset \{1, \ldots, s\}$ is an arbitrary nonempty subset. Define $p = p_X := \{\underline{n} = (n_1, \ldots, n_s) \in \mathbb{N}^s | \sum_{i \in X} n_i \geq 1\}$. Obviously, we have $\mathbb{N}^s \setminus \{(0, \ldots, 0)\} = p_{\{1,\ldots,s\}}$. This is again the unique maximal ideal. The generic point of $(\mathbb{N}^s, +)$ is again the empty set. The minimal prime ideals in $\operatorname{Spec} \mathbb{N}^r$ different from the generic points are given as $p = p_X$ with $X = \{i\}, i = 1, \ldots, s$. Explicitly we have in these cases $p_i = \{\underline{n} = (n_1, \ldots, n_s) \in \mathbb{N}^s | n_i \geq 1\}$ for $i = 1, \ldots, s$.

Proposition 1.9. *One has* $\operatorname{Spec} \mathbb{N}^s = \{p_X | X \subseteq \{1, \ldots, s\}\}$.

Proof. Exercise. □

It is easy to extend these definitions to arbitrary $\mathbb{F}_1$-schemes. This is done as usual by glueing.

Definition 1.10.
- An $\mathbb{F}_1$-scheme is a topological space X together with a sheaf $\mathcal{M}_X$ of monoids such that for any $x \in X$ there is an open neighbourhood $U \subseteq X$ such that $(U, \mathcal{M}_X|_U)$ is isomorphic to an affine $\mathbb{F}_1$-scheme, given as $(M, \cdot)^{\tilde{}}$ for an appropriate monoid $(M, \cdot)$.
- A morphism of schemes over $\mathbb{F}_1$ is a local morphism of "monoided" spaces.

For details on these definitions, see [4].

Finally we construct the s-dimensional projective space $\mathbb{P}^s$ over $\mathbb{F}_1$ by glueing s-dimensional affine spaces $\mathbb{A}^s$. As in the classical situation one obtains $s + 1$ copies of affine s-space as $\mathbb{P}^s \setminus \{z = (z_0, \ldots, z_s) | z_i = 0\} = \mathbb{A}^s_{(i)} \cong \mathbb{A}^s$ for $i = 0, \ldots, s$.

In particular, we have the affine spaces $\mathbb{A}^s_{(s)} = \operatorname{Spec} \mathbb{N}^s$ and $\mathbb{A}^s_{(s-1)} = \operatorname{Spec} \mathbb{N}^s$, where the glueing is given between the open parts

$$\mathbb{A}^s_{(s,s-1)} = \operatorname{Spec} \mathbb{N}^s[(0, 1, \ldots, 0, 1)] \simeq \operatorname{Spec}(\mathbb{N}^{s-1} \oplus \mathbb{Z})$$

and

$$\mathbb{A}^s_{(s-1,s)} = \operatorname{Spec} \mathbb{N}^s[(0, \ldots, 0, 1)] \simeq \operatorname{Spec}(\mathbb{N}^{s-1} \oplus \mathbb{Z})$$

of $\mathbb{A}^s_{(s)}$ resp. $\mathbb{A}^s_{(s-1)}$ as

$$\psi_{s,s-1} : \mathbb{A}^s_{(s,s-1)} \to \mathbb{A}^s_{(s-1,s)}$$

or on the associated monoids as

$$\tilde{\psi}_{s,s-1} : \mathbb{N}^s[(0,\ldots,0,1)] \to \mathbb{N}^s[(0,\ldots,0,1)],\ (m_1,\ldots,m_s) \mapsto (n_1,\ldots,n_s)$$

where we have $n_i = m_i$ for $i = 1,\ldots,s-1$ and $n_s = -(m_1 + \ldots + m_s)$. The "hyperplane at infinity", H_∞, given classically as $\mathbb{P}^s \setminus \mathbb{A}^s_{(s)}$, is given on the open part $\mathbb{A}^s_{(s-1)}$ of $\mathbb{P}^s$ as $V(p)$ for the associated minimal prime ideal $p = \{(m_1,\ldots,m_s) \in \mathbb{N}^s | m_s \geq 1\}$.

Denoting η the generic point of $\mathbb{A}^s_{(s)}$ and $\mathcal{O}_{\mathbb{A}^s_{(s-1)},\eta} = \mathbb{Z}^s$ for the corresponding local ring (the substitute for the field of rational functions in our context), we have the canonical homomorphism of local rings, that is, monoids

$$j : \mathcal{O}_{\mathbb{A}^s_{(s-1),p}} = \mathbb{Z}^{s-1} \oplus \mathbb{N} \hookrightarrow \mathcal{O}_{\mathbb{A}^s_{(s-1),\eta}}$$

which is here the obvious embedding. Using the glueing isomorphism above, we obtain the diagram

$$\begin{tikzcd}
\mathcal{O}(\mathbb{A}^s_{(s)}) = \mathbb{N}^s \arrow[dr] & & \mathcal{O}_{\mathbb{P}^s,p} = \{(n_1,\ldots,n_s) \in \mathbb{Z}^s | \sum_{i=1}^{s} n_i \leq 0\} \arrow[dl] \\
& \mathcal{O}_{\mathbb{P}^s,\eta} = \mathbb{Z}^s &
\end{tikzcd}$$

(where we have used the relation $n_1 + \cdots + n_s = -m_s \leq 0$)

2. Vector bundles over projective space.

Definition 2.1. A module over a monoid $(M,\cdot)$ is a set E together with an action of M on E in the usual sense. In particular $1 \in M$ acts as identity.

As explained in [4] one associates with such a module E a sheaf $E\tilde{}$ of modules on the affine scheme $\operatorname{Spec} M$, (that is, a sheaf of sets over the sheaf of monoids $M\tilde{}$).

Definition 2.2. A quasicoherent sheaf of modules $\mathcal{E}$ on the $\mathbb{F}_1$-scheme X is a sheaf of modules on the monoided space X such that there exists an open affine cover $(U_i; i \in I)$ of $X, U_i = \operatorname{Spec} M_i$ with monoids M_i and M_i-modules E_i such that $E_i\tilde{} \simeq (\mathcal{E}|_{U_i}),\ i \in I$ holds for all the restrictions.

See again [4] for details.

Definition 2.3. Let $(M,\cdot)$ be a monoid.

(i) The direct sum of two M-modules $E,\ E'$ is the disjoint union $E \mathbin{\dot\cup} E'$.
(ii) The direct sum for sheaves of modules is defined correspondingly.

For any monoid $(M,\cdot)$ one has the free module of rank one $E = M$ with the monoid action on it. Similarly one has the free module of rank r on $(M,\cdot)$, given as the disjoint union of r copies of $M, E = M \mathbin{\dot\cup} M \mathbin{\dot\cup} \cdots \mathbin{\dot\cup} M$ with the obvious M-action on it.

Definition 2.4. A sheaf of modules $\mathcal{E}$ over the scheme $\mathcal{M}$ is locally free of rank r iff there exists an open covering $(U_i, i \in I)$ of $\mathcal{M}$ by affine schemes $U_i = \operatorname{Spec} M_i$ such that $\mathcal{E}|U_i \simeq (M_i \mathbin{\dot\cup} \cdots \mathbin{\dot\cup} M_i))\tilde{}$, ($r$-times). An invertible sheaf or line bundle is a locally free sheaf of rank 1.

Remarks 2.5. (i) The terms 'locally free sheaf' and 'vector bundle' are used here synonymously.

(ii) As in the classical situation, there are line bundles (or invertible sheaves) $\mathcal{O}(n)$. These can be described as follows: For U open, $U \subset \mathbb{A}^s$, one has $\mathcal{O}(n)(U) := \mathcal{O}(U))$, for U open, $U \not\subset \mathbb{A}^s$, one has

$$\mathcal{O}(n)(U) := \mathcal{O}(U \cap \mathbb{A}^s) \cap \left\{ (x_1, \ldots, x_s) \in \mathbb{Z}^s \mid \sum_{i=1}^{s} x_i \leq n \right\}.$$

One can check immediately, that by this definition $\mathcal{O}(n)$ is an invertible sheaf of modules over $\mathbb{P}^s$.

$n := \deg(\mathcal{O}(n)$ is denoted as usual as the degree of the invertible sheaf $\mathcal{O}(n)$.

Theorem 2.6. (i) *Any line bundle on projective s-space $\mathbb{P}^s$ is of the form $\mathcal{O}(n)$.*

(ii) *Any locally free sheaf $\mathcal{E}$ of rank r on $\mathbb{P}^s$ is a direct sum of invertible sheaves $\mathcal{O}(n_1), \ldots, \mathcal{O}(n_r)$, so that*

$$\mathcal{E} \simeq \mathcal{O}(n_1) \oplus \cdots \oplus \mathcal{O}(n_r)$$

The numbers $n_1, \cdots, n_r \in \mathbb{Z}$ are determined uniquely up to permutation.

Proof. We show immediately (ii), upon discussing this, (i) will follow. We consider the diagram for $\mathbb{P}^s$ described in the first section, namely

$$\begin{tikzcd}[column sep=small] \mathrm{Spec}(\mathbb{N}^s) = \mathbb{A}^s_{(s)} \arrow[dr] & & \mathrm{Spec}(\mathcal{O}_{\mathbb{P}^s,p}) \arrow[dl] \\ & \mathrm{Spec}(\mathcal{O}_{\mathbb{P}^s,\eta}) & \end{tikzcd}$$

and the restriction of our given locally free sheaf $\mathcal{E}$ to this diagram. We obtain a diagram of the following type:

$$\begin{tikzcd}[column sep=small] \mathbb{N}^s \,\dot{\cup} \cdots \dot{\cup}\, \mathbb{N}^s \arrow[dr] & & \mathcal{E}_p \arrow[dl] \\ & \mathbb{Z}^s \,\dot{\cup} \cdots \dot{\cup}\, \mathbb{Z}^s & \end{tikzcd}$$

Here, one should remark that the sheaf of modules $\mathcal{E}|_{\mathbb{A}^s_{(s)}}$ is actually free.

In the classical context this is a deep theorem by Quillen and Suslin, proving a conjecture by Serre. Here, it is a mere triviality, as the monoid $\mathbb{N}^s$ is local with unique maximal ideal $\underline{m} = \{(n_1, \ldots, n_s) \in \mathbb{N}^s | \sum_{i=1}^{s} n_i \geq 1\}$ and the local ring $\mathbb{N}^s_{\underline{m}} \simeq \mathbb{N}^s$ canonically. Therefore, any locally free module E of rank r is actually free of rank r. Additionally, we can assume that j above is the canonical embedding. $j(\mathcal{E}_p) \subseteq (\mathbb{Z}^s \,\dot{\cup} \ldots \dot{\cup}\, \mathbb{Z}^s) = r \times \mathbb{Z}^s$ is a free module over the local ring $\mathcal{O}_{\mathbb{P}^s,p} = \{(n_1, \ldots, n_s) \in \mathbb{Z}^s | \sum_{i=1}^{s} n_i \leq 0\}$.

For each $i \in \{1, \ldots, s\}$ we consider $(i, \mathbb{Z}^s) \cap j(\mathcal{E}_p)$. Here $(i, \mathbb{Z}^s)$ denotes the ith copy of $\mathbb{Z}^s$ in $\mathbb{Z}^s \,\dot{\cup} \cdots \dot{\cup}\, \mathbb{Z}^s$ and j is the embedding mentioned above. This intersection is not empty, because upon localisation to the generic point, we obtain $r \times \mathbb{Z}^s$. So, $j(\mathcal{E}_p)$ cannot miss any of the s components. As $(i, \mathbb{Z}^s) \cap j(\mathcal{E}_p)$ obviously is a locally free $\mathcal{O}_{\mathbb{P}^s,p}$-module, one can find $(n_1^{(i)}, \dots, n_s^{(i)}) \in (i, \mathbb{Z}^s) \cap j(\mathcal{E}_p)$ such that

$$\begin{aligned}(i, \mathbb{Z}^s) \cap j(\mathcal{E}_p) &= (n_1^{(i)}, \dots, n_s^{(i)}) + \mathcal{O}_{\mathbb{P}^s,p}\\ &= \{(n_1^{(i)} + m_1, \dots, n_s^{(i)} + m_s) | \sum_{j=1}^{s} m_j \leq 0\}\end{aligned}$$

So, one can find the different generators by looking at the $(n_1^{(i)}, \dots, n_s^{(i)}) \in (i, \mathbb{Z}^s) \cap j(\mathcal{E}_p)$ such that $\sum_{j=1}^s n_j$ is maximal.

$\sum_{j=1}^s n_j^{(i)} =: \deg(L_i)$ is the only invariant of the submodule $(i, \mathbb{Z}^s) \cap j(\mathcal{E}_p)$. In toto we have shown the following three points:

1. Upon restriction to the diagram

$$\begin{tikzcd} \mathbb{A}^s_{(s)} \arrow[dr] & & \mathrm{Spec}(\mathcal{O}_{\mathbb{P}^s,p}) \arrow[dl] \\ & \mathrm{Spec}(\mathcal{O}_{\mathbb{P}^s,\eta}) & \end{tikzcd}$$

   a line bundle $\mathcal{L}$ is given by a diagram of modules

$$\begin{tikzcd} \mathbb{N}^s \arrow[dr] & & (d, 0, \dots, 0) + \mathcal{O}_{\mathbb{P},p} \arrow[dl] \\ & \mathbb{Z}^s & \end{tikzcd}$$

   where $d = \deg(\mathcal{L})$ is the degree of the invertible sheaf, which is an invariant. We denote $\mathcal{L} = \mathcal{O}(d)$.
2. One has a decomposition

$$\mathcal{E} = \bigoplus_{i=1}^{r} \mathcal{O}(d_i)$$

   into a direct sum of invertible sheaves.
3. The degrees $d_i (i = 1, \dots, r)$ of the direct summands are uniquely determined as are the summands $\mathcal{O}(d_i)$ themselves, up to permutation.

We have even shown more, namely that any reflexive sheaf of modules is a direct sum of invertible sheaves. In particular, such a sheaf is locally free, which is of course different from the classical situation. □

Remark 2.7. As indicated to us by the referee of the paper, upon comparing our result with the classical situation one should probably not think of general vector bundles on $\mathbb{P}^n$ but see the result in the context of toric geometry. As can be seen in [1] and [4], using the canonical morphism

$$\mathrm{Spec}(\mathbb{Z}) \to \mathrm{Spec}(\mathbb{F}_1)$$

(or for simplicity

$$\mathrm{Spec}(\mathbb{C}) \to \mathrm{Spec}(\mathbb{F}_1))$$

the pull-back of a smooth variety over $\mathbb{F}_1$ will be a smooth toric variety over $\mathrm{Spec}(\mathbb{Z})$ (or simpler over $\mathrm{Spec}\,\mathbb{C}$). Similarly, as indicated in particular in [1], there is also a pull-back operation for quasicoherent sheaves, which gives quasi-coherent sheaves over toric varieties (say over $\mathbb{C}$), equipped with an action of the torus involved on the quasi-coherent sheaf of modules, compatible with the action of the torus on the variety. As we do not want to go into greater detail here, we just describe the situation from the other side. So we start with a smooth toric variety X over the field $\mathbb{C}$ of complex numbers. The action of the torus T on X is part of the data given. So, in particular, we have a dense open orbit $X' = T \cdot x_0 \simeq T \subset X$. Furthermore, we assume, that $\mathcal{E}$ is a locally free sheaf on X, such that T acts on $\mathcal{E}$, compatible with the action on X. Additionally, we can assume, that $X \setminus X' = \bigcup_{i=1}^{m} D_i$, a union of irreducible divisors in X and again T is acting on each of the D_i. In particular, the generic points $\xi_i \in D_i$ are fixed under the action of T.

Then we have for the ring of sections

$$\mathcal{O}(X') \cong \mathcal{O}[T] \cong \mathbb{C}[t_i, t_i^{-1} \mid i = 1, \ldots, s\}$$

and in it the monoid

$$\mathbb{Z}^s = \langle t_i, t_i^{-1} \mid i = 1, \ldots, s\rangle$$

generated by the characters $\chi_i(t) = t_i$ of T.

Because of the compatibility of the action of T on X as well as X' and also on $\mathcal{E}$, we can describe the $\mathcal{O}(X')$-module of sections $\mathcal{E}(X')$ as a direct sum of eigenspaces. Keeping book of the set of characters of T, acting on these eigenspaces, we come exactly to a (reflexive) sheaf of modules over the $\mathbb{F}_1$-space underlying X in the sense of our paper.

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